

# Aspects of Hořava-Lifshitz cosmology

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We review some general aspects of Hořava-Lifshitz cosmology. Formulating it in its basic version, we extract the cosmological equations and we use observational data in order to constrain the parameters of the theory. Through a phase-space analysis we extract the late-time stable solutions, and we show that eternal expansion, and bouncing and cyclic behavior can arise naturally. Concerning the effective dark energy sector we show that it can describe the phantom phase without the use of a phantom field. However, performing a detailed perturbation analysis, we see that Hořava-Lifshitz gravity in its basic version suffers from instabilities. Therefore, suitable generalizations are required in order for this novel theory to be a candidate for the description of nature.

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## I. INTRODUCTION

Almost one year ago Hořava proposed a power-counting renormalizable theory with consistent ultra-violet (UV) behavior [1–3]. Although presenting an infrared (IR) fixed point, namely General Relativity, in the UV the theory exhibits an anisotropic, Lifshitz scaling between time and space. Due to these novel features, there has been a large amount of effort in examining and extending the properties of the theory itself [4–11]. Additionally, application of Hořava-Lifshitz gravity as a cosmological framework gives rise to Hořava-Lifshitz cosmology, which proves to lead to interesting behavior [12]. In particular, one can examine specific solution subclasses [13–15], the phase-space behavior [16–18], the gravitational wave production [19], the perturbation spectrum [20–22], the matter bounce [23–26], the black hole properties [27–29], the dark energy phenomenology [30–33], the observational constraints on the parameters of the theory [34–36], the astrophysical phenomenology [37], the thermodynamic properties [38, 39] etc. However, despite this extended research, there are still many ambiguities if Hořava-Lifshitz gravity is reliable and capable of a successful description of the gravitational background of our world, as well as of the cosmological behavior of the universe [5–7, 11, 40, 41].

In the present work we review the basic aspects of Hořava-Lifshitz cosmology. The manuscript is organized as follows: In section II we present the simple version of Hořava-Lifshitz cosmology, in both its detailed-balance and beyond-detailed-balance version. In section III we use observational data in order to constrain the parameters of the theory. In section IV we present the results of the phase-space analysis, in V we present the bouncing and cyclic solutions, and in VI we extend the theory in order to present a more realistic dark energy phenomenology. In section VII, through a perturbation analysis, we

discuss the instabilities of the simple versions of the theory, and thus in section VIII we present a healthy extension of Hořava-Lifshitz gravity. Finally, section IX is devoted to the summary of our results.

## II. HOŘAVA-LIFSHITZ COSMOLOGY

In this section we briefly review the scenario where the cosmological evolution is governed by the simple version of Hořava-Lifshitz gravity [12]. The dynamical variables are the lapse and shift functions,  $N$  and  $N_i$  respectively, and the spatial metric  $g_{ij}$  (roman letters indicate spatial indices). In terms of these fields the full metric is written as:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \quad (1)$$

and the scaling transformation of the coordinates reads as  $t \rightarrow l^3 t$  and  $x^i \rightarrow l x^i$ .

### A. Detailed Balance

The gravitational action is decomposed into a kinetic and a potential part as  $S_g = \int dt d^3x \sqrt{g} N (\mathcal{L}_K + \mathcal{L}_V)$ . The assumption of detailed balance [3] reduces the possible terms in the Lagrangian, and it allows for a quantum inheritance principle [1], since the  $(D+1)$ -dimensional theory acquires the renormalization properties of the  $D$ -dimensional one. Under the detailed balance condition the full action of Hořava-Lifshitz gravity is given by

$$\begin{aligned} S_g = & \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) \right. \\ & + \frac{\kappa^2}{2w^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu}{2w^2} \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R_k^l + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} \\ & \left. - \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left[ \frac{1 - 4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right] \right\}, \quad (2) \end{aligned}$$

where  $K_{ij} = (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) / 2N$  is the extrinsic curvature and  $C^{ij} = \epsilon^{ijk} \nabla_k (R_i^j - R \delta_i^j / 4) / \sqrt{g}$  the Cotton tensor, and the covariant derivatives are defined with

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respect to the spatial metric  $g_{ij}$ .  $\epsilon^{ijk}$  is the totally antisymmetric unit tensor,  $\lambda$  is a dimensionless constant and the variables  $\kappa$ ,  $w$  and  $\mu$  are constants. Finally, we mention that in action (2) we have already performed the usual analytic continuation of the parameters  $\mu$  and  $w$  of the original version of Hořava-Lifshitz gravity, since such a procedure is required in order to obtain a realistic cosmology [13, 14, 28, 38] (although it could fatally affect the gravitational theory itself).

In order to add the matter component we follow the hydrodynamical approach of adding a cosmological stress-energy tensor to the gravitational field equations, by demanding to recover the usual general relativity formulation in the low-energy limit [6, 16, 32]. Thus, this matter-tensor is a hydrodynamical approximation with  $\rho_m$  and  $p_m$  (or  $\rho_m$  and  $w_m$ ) as parameters. Similarly, one can additionally include the standard-model-radiation component, with the additional parameters  $\rho_r$  and  $w_r$ .

In order to investigate cosmological frameworks, we impose the projectability condition [5] and we use an FRW metric

$$N = 1, \quad g_{ij} = a^2(t)\gamma_{ij}, \quad N^i = 0, \quad (3)$$

with

$$\gamma_{ij}dx^i dx^j = \frac{dr^2}{1 - Kr^2} + r^2 d\Omega_2^2, \quad (4)$$

where  $K <, =, > 0$  corresponding to open, flat, and closed universe respectively. By varying  $N$  and  $g_{ij}$ , we extract the Friedmann equations:

$$\begin{aligned} H^2 &= \frac{\kappa^2}{6(3\lambda - 1)}(\rho_m + \rho_r) + \\ &+ \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2\mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \\ &- \frac{\kappa^4\mu^2 \Lambda K}{8(3\lambda - 1)^2 a^2}, \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{H} + \frac{3}{2}H^2 &= -\frac{\kappa^2}{4(3\lambda - 1)}(w_m\rho_m + w_r\rho_r) - \\ &- \frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2\mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \\ &- \frac{\kappa^4\mu^2 \Lambda K}{16(3\lambda - 1)^2 a^2}, \end{aligned} \quad (6)$$

where  $H \equiv \frac{\dot{a}}{a}$  is the Hubble parameter. As usual,  $\rho_m$  follows the standard evolution equation  $\dot{\rho}_m + 3H(\rho_m + p_m) = 0$ , while  $\rho_r$  follows  $\dot{\rho}_r + 3H(\rho_r + p_r) = 0$ . Finally, concerning the dark-energy sector we can define

$$\rho_{DE} \equiv \frac{3\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2\mu^2 \Lambda^2}{8(3\lambda - 1)} \quad (7)$$

$$p_{DE} \equiv \frac{\kappa^2\mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2\mu^2 \Lambda^2}{8(3\lambda - 1)}. \quad (8)$$

The term proportional to  $a^{-4}$  is the usual “dark radiation term”, present in Hořava-Lifshitz cosmology [12], while the constant term is just the explicit cosmological constant. Therefore, in expressions (7),(8) we have defined the energy density and pressure for the effective dark energy, which incorporates the aforementioned contributions. Note that using (7),(8) it is straightforward to show that these dark energy quantities satisfy the standard evolution equation:  $\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0$ .

If we require expressions (5) to coincide with the standard Friedmann equations, in units where  $c = 1$  we set [12]:

$$\begin{aligned} G_{\text{cosmo}} &= \frac{\kappa^2}{16\pi(3\lambda - 1)} \\ \frac{\kappa^4\mu^2 \Lambda}{8(3\lambda - 1)^2} &= 1, \end{aligned} \quad (9)$$

where  $G_{\text{cosmo}}$  is the “cosmological” Newton’s constant, that is the one that is read from the Friedmann equations. We mention that in theories with Lorentz invariance breaking  $G_{\text{cosmo}}$  does not coincide with the “gravitational” Newton’s constant  $G_{\text{grav}}$ , that is the one that is read from the action, unless Lorentz invariance is restored [42]. For completeness we mention that in our case  $G_{\text{grav}} = \kappa^2/(32\pi)$ , as it can be straightforwardly read from the action (2). Thus, it becomes obvious that in the IR ( $\lambda = 1$ ), where Lorentz invariance is restored,  $G_{\text{cosmo}}$  and  $G_{\text{grav}}$  coincide.

## B. Beyond Detailed Balance

The aforementioned formulation of Hořava-Lifshitz cosmology has been performed under the imposition of the detailed-balance condition. However, in the literature there is a discussion whether this condition leads to reliable results or if it is able to reveal the full information of Hořava-Lifshitz gravity [12]. Therefore, one needs to investigate also the Friedman equations in the case where detailed balance is relaxed. In such a case one can in general write [5–7, 16, 17]:

$$\begin{aligned} H^2 &= \frac{2\sigma_0}{(3\lambda - 1)}(\rho_m + \rho_r) + \\ &+ \frac{2}{(3\lambda - 1)} \left[ \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \right] + \\ &+ \frac{\sigma_2}{3(3\lambda - 1)} \frac{K}{a^2} \end{aligned} \quad (10)$$

$$\begin{aligned} \dot{H} + \frac{3}{2}H^2 &= -\frac{3\sigma_0}{(3\lambda - 1)}(w_m\rho_m + w_r\rho_r) - \\ &- \frac{3}{(3\lambda - 1)} \left[ -\frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6} \right] + \\ &+ \frac{\sigma_2}{6(3\lambda - 1)} \frac{K}{a^2}, \end{aligned} \quad (11)$$

where  $\sigma_0 \equiv \kappa^2/12$ , and the constants  $\sigma_i$  are arbitrary (with  $\sigma_2$  being negative and  $\sigma_4$  positive). Furthermore, the dark-energy quantities are generalized to

$$\rho_{DE}|_{\text{non-db}} \equiv \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \quad (12)$$

$$p_{DE}|_{\text{non-db}} \equiv -\frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6}. \quad (13)$$

Again, it is easy to show that

$$\dot{\rho}_{DE}|_{\text{non-db}} + 3H(\rho_{DE}|_{\text{non-db}} + p_{DE}|_{\text{non-db}}) = 0. \quad (14)$$

Finally, if we force (10),(11) to coincide with the standard Friedmann equations, we obtain:

$$G_{\text{cosmo}} = \frac{6\sigma_0}{8\pi(3\lambda - 1)} \\ \sigma_2 = -3(3\lambda - 1), \quad (15)$$

while in this case the “gravitational” Newton’s constant  $G_{\text{grav}}$  writes as  $G_{\text{grav}} = 6\sigma_0/(16\pi)$ . Similarly to the detailed balance case, in the IR ( $\lambda = 1$ )  $G_{\text{cosmo}}$  and  $G_{\text{grav}}$  coincide.

### III. OBSERVATIONAL CONSTRAINTS

Having presented the cosmological equations of a universe governed by Hořava-Lifshitz gravity, both with and without the detailed-balance condition, we now proceed to study the observational constraints on the model parameters [34, 35].

#### A. Constraints on Detailed-Balance scenario

We work in the usual units suitable for observational comparisons, namely setting  $8\pi G_{\text{grav}} = 1$  (we have already set  $c = 1$  in order to obtain (9)). This allows us to reduce the parameter space, since in this case  $G_{\text{grav}}$  gives  $\kappa^2 = 4$  and thus (9) lead to:  $G_{\text{cosmo}} = \frac{1}{4\pi(3\lambda-1)}$  an  $\mu^2\Lambda = \frac{(3\lambda-1)^2}{2}$ . In order to proceed to the elaboration of observational data, we consider as usual the matter (dark plus baryonic) component to be dust, that is  $w_m \approx 0$ , and similarly for the standard-model radiation we consider  $w_r = 1/3$ , where both assumptions are valid in the epochs in which observations focus. Therefore, the corresponding evolution equations give  $\rho_m = \rho_{m0}/a^3$  and  $\rho_r = \rho_{r0}/a^4$  respectively. Additionally, instead of the scale factor it proves convenient to use the redshift  $z$  as the independent variable, which is given by  $1+z \equiv a_0/a = 1/a$ . Finally, we introduce the usual density parameters ( $\Omega_m \equiv \rho_m/(3H^2)$ ,  $\Omega_K \equiv -K/(H^2a^2)$ ,  $\Omega_r \equiv \rho_r/(3H^2)$ ). Inserting these relations into Friedmann equation (5) we acquire:

$$H^2 = H_0^2 \left\{ \frac{2}{(3\lambda-1)} \left[ \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 \right] + \Omega_{K0}(1+z)^2 + \left[ \omega + \frac{\Omega_{K0}^2}{4\omega}(1+z)^4 \right] \right\}, \quad (16)$$

where we have also introduced the dimensionless parameter  $\omega \equiv \frac{\Lambda}{2H_0^2}$ , and where a 0-subscript denotes the present value of the corresponding quantity. Applying this relation at present we get:

$$\frac{2}{(3\lambda-1)} (\Omega_{m0} + \Omega_{r0}) + \Omega_{K0} + \omega + \frac{\Omega_{K0}^2}{4\omega} = 1. \quad (17)$$

We remind that the term  $\Omega_{K0}^2/(4\omega)$  is the coefficient of the dark radiation term, which is a characteristic feature of the Hořava-Lifshitz gravitational background. Since this dark radiation component has been present also during the time of nucleosynthesis, it is subject to bounds from Big Bang Nucleosynthesis (BBN). As discussed in more details in the Appendix of [34], if the upper limit on the total amount of dark radiation allowed during BBN is expressed through the parameter  $\Delta N_\nu$  of the effective neutrino species [43], then we obtain the following constraint:

$$\frac{\Omega_{K0}^2}{4\omega} = 0.135\Delta N_\nu\Omega_{r0}. \quad (18)$$

In summary, the scenario at hand involves four parameters (we fix  $H_0$  by its 7-year WMAP best-fit values, given in Table 1 of [44]), namely  $\Omega_{m0}$ ,  $\Omega_{K0}$ ,  $\omega$  and  $\Delta N_\nu$ , subject to constraint equations (17) and (18). We marginalize over the cosmological parameters  $\Omega_{m0}$ ,  $\Omega_{b0}$ ,  $\Omega_{r0}$  and  $H_0$ . Of the four remaining parameters, only two are independent, and we choose  $\lambda$  and  $\Delta N_\nu$  as our free parameters. Once these are chosen, and for a given choice of curvature,  $\Omega_{K0}$  and  $\omega$  are immediately fixed from the constraint equations. In particular,  $\omega$  can be determined by eliminating  $\Omega_{K0}$  from relations (17) and (18):

$$\omega - 2 \operatorname{sgn}(\Omega_{K0}) \sqrt{0.135\Delta N_\nu \Omega_{r0} \omega} + 0.135\Delta N_\nu \Omega_{r0} + 2 \left[ \frac{\Omega_{m0} + \Omega_{r0}}{3\lambda - 1} \right] - 1 = 0. \quad (19)$$

$\Omega_{K0}$  can then be found from  $\omega$  using (18).

In Fig. 1 we use a combination of observational data from SNIa, BAO and CMB to construct likelihood contours for the parameters  $\Omega_{m0}$  and  $\Delta N_\nu$  for positive curvature. Additionally, in Fig. 2 we display the likelihood-contours for the free parameters  $\lambda$  vs  $\Delta N_\nu$  for positive curvature, where all other parameters have been marginalized over. Finally, in Table I we summarize the  $1\sigma$  limits on the parameter values for the detailed-balance scenario.

$K$	$\kappa^2/(8\pi G_{\text{grav}})$	$(1/H_0^2)\Lambda$	$(8\pi G_{\text{grav}}H_0)\mu$	$\lambda$	$\Delta N_\nu$
$>0$	4	(0, 1.46)	(1.37, $\infty$ )	(0.98, 1.01)	(0, 0.32)
$<0$	4	(0, 1.46)	(1.8, $\infty$ )	(0.97, 1.01)	(0, 0.68)

TABLE I:  $1\sigma$  limits on the parameter values for the detailed-balance scenario, for positive and negative curvature. The cosmological parameters  $\Omega_{m0}$ ,  $\Omega_{b0}$ ,  $\Omega_{r0}$  and  $H_0$  have been marginalized over.

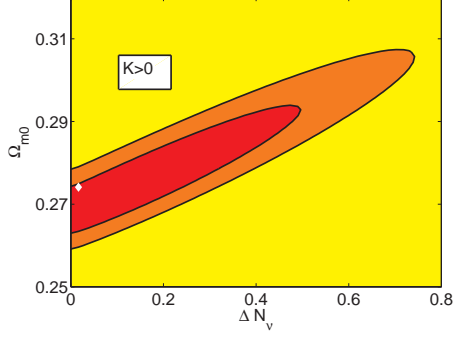


FIG. 1: (Color Online)  $1\sigma$  and  $2\sigma$  contour plots of  $\Omega_{m0}$  vs  $\Delta N_\nu$  for positive curvature ( $K > 0$ ), under SNIa, BAO and CMB observational data. The white diamond marks the best-fit point. The model parameters  $\omega \equiv \Lambda/(2H_0^2)$  and  $\Omega_{K0} \equiv -K/(H_0^2)$  are related to  $\Omega_{m0}$  and  $\Delta N_\nu$  through (18) and (19).

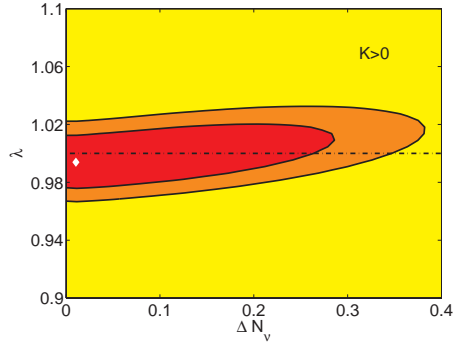


FIG. 2: (Color online)  $1\sigma$  and  $2\sigma$  contour plots of  $\lambda$  vs  $\Delta N_\nu$  for positive ( $K > 0$ ) curvature, in the detailed-balance scenario, under SNIa, BAO and CMB observational data. The remaining parameters have been marginalized over.

In conclusion, we see that the Hořava-Lifshitz cosmological scenario under the detailed balance condition is not ruled out by observations. However, there are tight constraints on the model parameters. Furthermore, the data constrain  $\lambda$  to roughly  $\lambda = 1^{+0.01}_{-0.02}$  at the  $1\sigma$  level, that is to a very narrow window around its IR value, while its best fit value is very close to 1 ( $\lambda_{b,f} = 0.006$ ).

### B. Constraints on Beyond-Detailed-Balance scenario

In units where  $8\pi G_{\text{grav}} = 1$ ,  $G_{\text{grav}}$  gives  $\sigma_0 = 1/3$ . Following the procedure of the previous subsection, the Friedmann equation (10) can be written as

$$H^2 = H_0^2 \left\{ \frac{2}{(3\lambda - 1)} \left[ \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 \right] + \Omega_{K0}(1+z)^2 + \frac{2}{(3\lambda - 1)} \left[ \omega_1 + \omega_3(1+z)^4 + \omega_4(1+z)^6 \right] \right\},$$

where we have introduced the dimensionless parameters  $\omega_1 = \frac{\sigma_1}{6H_0^2}$ ,  $\omega_3 = \frac{\sigma_3 H_0^2 \Omega_{K0}^2}{6}$  and  $\omega_4 = -\frac{\sigma_4 \Omega_{K0}}{6}$ . Additionally, we consider the combination  $\omega_4$  to be positive, in order to ensure that the Hubble parameter is real for all redshifts.

In summary, the present scenario involves the following parameters: the cosmological parameters  $H_0$ ,  $\Omega_{m0}$ ,  $\Omega_{K0}$ ,  $\Omega_{b0}$ ,  $\Omega_{r0}$ , and the model parameters  $\lambda$ ,  $\omega_1$ ,  $\omega_3$  and  $\omega_4$ . Similarly to the detailed-balance section these are subject to two constraints. The first one arises from the Friedman equation at  $z = 0$ , which leads to

$$\frac{2}{(3\lambda - 1)} \left[ \Omega_{m0} + \Omega_{r0} + \omega_1 + \omega_3 + \omega_4 \right] + \Omega_{K0} = 1. \quad (20)$$

This constraint eliminates the parameter  $\omega_1$ . The second one arises from BBN considerations, since, as we show in the Appendix of [34], at the time of BBN ( $z = z_{\text{BBN}}$ ) we acquire [43]:

$$\omega_3 + \omega_4 (1 + z_{\text{BBN}})^2 = \omega_{3\text{max}} \equiv 0.135 \Delta N_\nu \Omega_{r0}, \quad (21)$$

where  $\omega_{3\text{max}}$  denotes the upper limit on  $\omega_3$ . In the following, we use expression (21) to eliminate  $\omega_4$ . For convenience, instead of  $\omega_3$  we define the new parameter  $\alpha \equiv \frac{\omega_3}{\omega_{3\text{max}}}$  [35].

We use relation (21) to eliminate  $\omega_4$  in favor of  $\alpha$  and  $\Delta N_\nu$ , and treat  $\lambda$ ,  $\alpha$ ,  $\Omega_{K0}$  and  $\Delta N_\nu$  as our free parameters, marginalizing over  $H_0$ ,  $\Omega_{m0}$ ,  $\Omega_{b0}$  and  $\Omega_{r0}$ . Using the combined SNIa+CMB+BAO data, we construct likelihood contours for different combinations of the above parameters. Figure 3 depicts the  $1\sigma$  and  $2\sigma$   $\omega_3 - |\Omega_{K0}|$  contours, for  $\Delta N_\nu = 2$ , for positive curvature, while Fig. 4 depicts the  $\lambda$ -variation. The approximate  $1\sigma$  limits

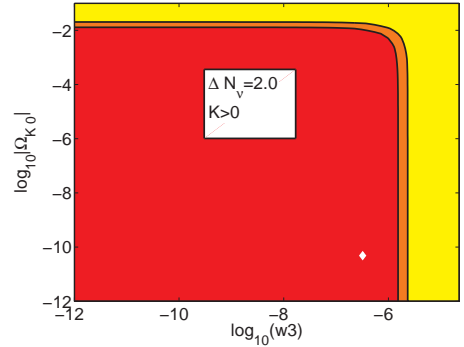


FIG. 3: (Color Online)  $1\sigma$  and  $2\sigma$  contour plots of  $\log_{10}(w_3)$  vs  $\log_{10}|\Omega_{K0}|$  for  $K > 0$  and  $\Delta N_\nu = 2.0$ , using SNIa, BAO and CMB data. The white diamond marks the best-fit point.

on the model parameters  $\sigma_i$  are presented in Table II. Additionally, in Table III we focus on the  $1\sigma$  limits of  $\alpha$  and  $\lambda$ .

$\sigma_0/(8\pi G)$	$\Delta N_\nu$	$\Omega_{K0}$	$(8\pi G/H_0^2) \sigma_1$	$\sigma_2$	$(8\pi G H_0^2) \sigma_3$	$\sigma_4/(8\pi G)$
1/3	0.1	(0, 0.01)	(4.29, 4.33)	-6	(0, 0.03)	$(-9.08 \times 10^{-22}, 0)$
1/3	0.1	(-0.01, 0)	(4.40, 4.45)	-6	(0, 0.81)	$(0, 5.66 \times 10^{-22})$
1/3	2.0	(0, 0.04)	(4.13, 4.45)	-6	(0, 0.01)	$(-1.77 \times 10^{-20}, -2.62 \times 10^{-21})$
1/3	2.0	(-0.01, 0)	(4.40, 4.45)	-6	(0, 0.23)	$(-2.61 \times 10^{-20}, -1.16 \times 10^{-20})$

TABLE II:  $1\sigma$  limits on the parameter values for the beyond-detailed-balance scenario, for positive and negative curvature, and for two values of the effective neutrino species parameter  $\Delta N_\nu$  (see text).

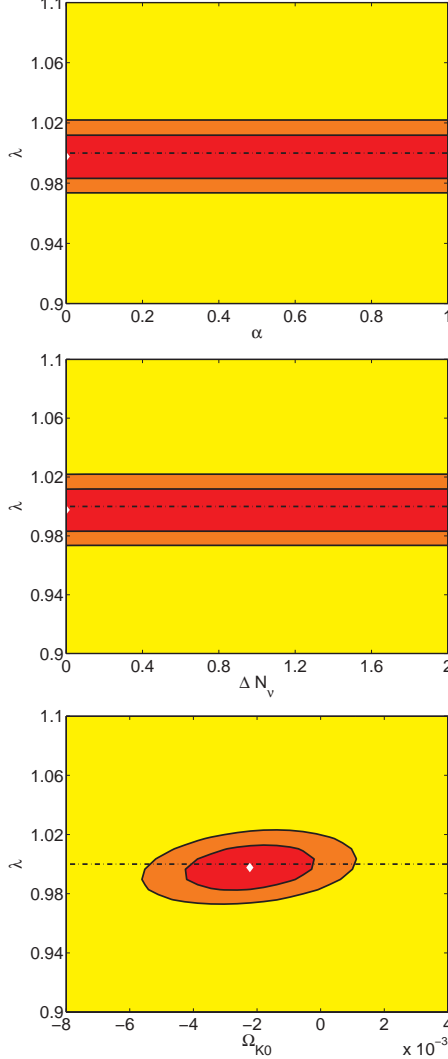


FIG. 4: (Color online) *Contour plots of different pairs of free parameters in the beyond-detailed-balance scenario, under SNIa, BAO and CMB observational data. In each case the parameters not included in the plots have been marginalized over. Color scheme as in Fig. 2.*

As we observe, in  $1\sigma$  confidence the running parameter  $\lambda$  of Hořava-Lifshitz gravity is restricted to the interval  $|\lambda - 1| \lesssim 0.02$ , for the entire allowed range of  $\omega_3$  (that is of  $\sigma_3$ ). Finally, the best fit value for  $\lambda$  restricts  $|\lambda - 1|$  to much more smaller values, namely  $|\lambda_{b.f} - 1| \approx 0.002$ .

$\Omega_{K0}$	$\Delta N_\nu$	$\alpha$	$\lambda$
$(-0.01, 0.01)$	$(0, 2)$	$(0, 1)$	$(0.98, 1.01)$

TABLE III:  $1\sigma$  limits on the free parameters of the beyond-detailed-balance scenario. The cosmological parameters  $\Omega_{m0}$ ,  $\Omega_{b0}$ ,  $\Omega_{r0}$  and  $H_0$  have been marginalized over.

#### IV. PHASE-SPACE ANALYSIS OF HOŘAVA-LIFSHITZ COSMOLOGY

In this section we review the results of the phase-space and stability analysis of Hořava-Lifshitz cosmology, with or without the detailed-balance condition, following [17]. We are interested in investigating the possible late-time solutions, and in these solutions we calculate various observable quantities, such as the dark-energy density and equation-of-state parameters.

We start by transforming the cosmological equations into an autonomous dynamical system [45], introducing suitable dimensionless variables which are combinations of the model variables and parameters. Then we extract the critical points of the autonomous system, and in order to determine their stability we linearize it around them and we examine the eigenvalues of the corresponding coefficient matrix of the perturbation equations.

In the case where the detailed-balance condition is imposed, we find that the universe can reach a bouncing-oscillatory state at late times, in which dark-energy, behaving as a simple cosmological constant, will be dominant. Such solutions arise purely from the novel terms of Hořava-Lifshitz cosmology, and in particular the dark-radiation term proportional to  $a^{-4}$  is responsible for the bounce, while the cosmological constant term is responsible for the turnaround.

In the case where the detailed-balance condition is abandoned, we find that the universe reaches an eternally expanding solution at late times, in which dark-energy, behaving like a cosmological constant, dominates completely. Note that according to the initial conditions, the universe on its way to this late-time attractor can be an expanding one with non-negligible matter content, independently of the specific form of the dark-matter content. These features make this scenario a good candidate for the description of our universe, in consistency with observations. Finally, in this case the universe has also a probability to reach an oscillatory solution at late times, if the initial conditions lie in its basin of attraction.

## V. BOUNCE AND CYCLIC BEHAVIOR

The possibility of late-time cyclic solutions that arose from the phase-space analysis, makes us to investigate it in more detail. Let us take a first look at how it is possible to obtain a cosmological bounce in this framework [46]. In the contracting phase we have  $H < 0$ , while in the expanding one we have  $H > 0$ , and by making use of the continuity equations it follows that at the bounce point  $H = 0$ . Throughout this transition  $\dot{H} > 0$ . On the other hand, for the transition from expansion to contraction, that is for the cosmological turnaround, we have  $H > 0$  before and  $H < 0$  after, while exactly on the turnaround point we have  $H = 0$ . Throughout this transition  $\dot{H} < 0$ .

The above conditions for a bounce and a turnaround can be easily fulfilled in Hořava-Lifshitz cosmology, as we observe from the two Friedmann equations (10) and (11). In particular, a cyclic scenario could be straightforwardly obtained if we consider a negative dark radiation term and a negative cosmological constant. During the expansion, the energy densities of all components decrease, which is not the case for the cosmological constant. Thus, its contribution will counterbalance that of dark matter, triggering a turnaround, after which the universe enters in the contracting phase. Then, after contraction to sufficiently small scale factors the dark radiation term will lead the universe to experience a bounce. Thus, the universe in such a model indeed presents a cyclic behavior, with a bounce and a turnaround at each cycle [25].

The absence of singularities in a cosmological scenario is a significant advantage. However, one must examine the proceeding of fluctuations through the bounce. In general, non-relativistic gravities, such is Hořava-Lifshitz one, are usually able to recover Einstein's general relativity as an emergent theory at low energy scales. Therefore, the cosmological fluctuations generated in this model should be consistent with those obtained in standard perturbation theory in the IR limit [23]. In particular, the perturbation spectrum presents a scale-invariant profile, if the universe has undergone a matter-dominated contracting phase [47]. However, the non-relativistic corrections in the Hořava-Lifshitz action could lead to a modification of the dispersion relations of perturbations. This issue has been addressed in [22] (see [20] and references therein for the perturbations of a pure expanding universe in Hořava-Lifshitz cosmology), which shows that the spectrum in the UV regime may have a red tilt in a bouncing universe. Moreover, the perturbation modes cannot enter the UV regime in the scenario of matter-bounce. Thus, the analysis of the cosmological perturbations in the IR regime is quite reliable.

## VI. A MORE REALISTIC HOŘAVA-LIFSHITZ DARK ENERGY

In section II we formulated Hořava-Lifshitz cosmology, in which one can define the effective dark energy sector

through (7),(8) in the detailed-balance case, or through (12),(13) in the beyond-detailed-balance case. Thus, one can straightforwardly obtain the dark-energy equation-of-state parameter in both cases, as  $w_{DE} = p_{DE}/\rho_{DE}$ . As can be immediately seen, in both cases  $w_{DE}$  lies above the phantom divide. However, according to observations,  $w_{DE}$  could have crossed  $-1$  in the recent cosmological past. Therefore, the question is whether we can formulate an extension of Hořava-Lifshitz cosmology, in which the dark energy equation-of-state parameter can experience the phantom-divide crossing.

For this shake we allow for an additional scalar field, which will contribute to the dark energy sector [30]<sup>1</sup>. Hence, we add a second scalar  $\sigma$ , with action

$$S_\sigma = \int dt d^3x \sqrt{g} N \left[ \frac{3\lambda - 1}{4} \frac{\dot{\sigma}^2}{N^2} + h_1 h_2 \sigma \nabla^2 \sigma - \frac{1}{2} h_2^2 \sigma \nabla^4 \sigma + \frac{1}{2} h_3^2 \sigma \nabla^6 \sigma - V(\sigma) \right], \quad (22)$$

where  $V(\sigma)$  accounts for the potential term of the  $\sigma$ -field and  $h_i$  are constants. Assuming homogeneity, that is  $\sigma \equiv \sigma(t)$ , its evolution equation will be given by

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{2}{3\lambda - 1} \frac{dV(\sigma)}{d\sigma} = 0. \quad (23)$$

Additionally, it can be easily seen that its contribution to the Friedmann equations of section II will be the standard scalar-field one, and thus one can absorb it in an extended dark energy sector, with energy density and pressure given by:

$$\begin{aligned} \rho_{DE} &\equiv \frac{3\lambda - 1}{4} \dot{\sigma}^2 + V(\sigma) + \frac{3\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \\ p_{DE} &\equiv \frac{3\lambda - 1}{4} \dot{\sigma}^2 - V(\sigma) + \frac{\kappa^2 \mu^2 K^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \end{aligned}$$

in the detailed-balance case, and by:

$$\begin{aligned} \rho_{DE}|_{\text{non-db}} &\equiv \frac{3\lambda - 1}{4} \dot{\sigma}^2 + V(\sigma) + \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{6a^4} + \frac{\sigma_4 K}{6a^6} \\ p_{DE}|_{\text{non-db}} &\equiv \frac{3\lambda - 1}{4} \dot{\sigma}^2 - V(\sigma) - \frac{\sigma_1}{6} + \frac{\sigma_3 K^2}{18a^4} + \frac{\sigma_4 K}{6a^6} \end{aligned}$$

in the beyond-detailed-balance one. Note that the dark energy density in both cases satisfies the usual conservation equation.

The aforementioned extended version of Hořava-Lifshitz dark energy can have a very interesting phenomenology. Firstly, the corresponding equation-of-state parameter  $w_{DE}$  can be above  $-1$ , below  $-1$ , or experience the  $-1$ -crossing during the cosmological evolution, as can be straightforwardly seen by the ratio  $w_{DE} = p_{DE}/\rho_{DE}$ . Thus, in this case, artifacts of Hořava-Lifshitz gravity could be detected through dark energy

<sup>1</sup> Note that one could alternatively generalize the gravitational action of Hořava-Lifshitz gravity itself [8, 32, 48].

observations. However, one still cannot distinguish between this model and alternative models that allow for the realization of  $w_{DE} < -1$  phase, such as modified gravity [49] or models with phantom [50] or quintom fields [51]. However, note that in the present formulation the additional scalar field is canonical, while in phantom and quintom scenarios the scalar field is phantom, and thus with ambiguous quantum behavior. The ability to describe the phantom phase and the phantom crossing with a canonical scalar field is a significant advantage of the scenario at hand, revealing the capabilities of Hořava-Lifshitz cosmology.

## VII. PERTURBATIVE INSTABILITIES IN HOŘAVA-LIFSHITZ GRAVITY

In the previous sections we showed the advantages of Hořava-Lifshitz cosmology at the background level. However, despite the capabilities of the scenario, our analysis does not enlighten the discussion about the possible conceptual problems and instabilities of Hořava-Lifshitz gravity, nor it can address the questions concerning the validity of its theoretical background. Thus, in this section we are interested in performing a detailed investigation of the gravitational perturbations of Hořava-Lifshitz gravity, using it as a tool to examine its consistency, studying both scalar and tensor sectors around a Minkowski background [7].

We consider coordinate transformations of the form  $x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \xi^\mu$ . Under this transformation the metric-perturbation around a given background changes as  $\delta\tilde{g}_{\mu\nu} = \delta g_{\mu\nu} - \nabla_\mu \xi_\nu - \nabla_\nu \xi_\mu$ . Therefore, the general perturbations of the metric (1) read:

$$\begin{aligned}\delta g_{00} &= -2a^2\phi \\ \delta g_{0i} &= a^2\partial_i B + a^2 Q_i \\ \delta g_{ij} &= a^2 h_{ij} - a^2(\partial_i W_j + \partial_j W_i) - 2a^2\psi\delta_{ij} + 2a^2\partial_i\partial_j E.\end{aligned}$$

The vector modes are assumed to be transverse, that is  $\partial_i W^i = \partial_i Q^i = 0$ , while the tensor mode is forced to be transverse and traceless:  $\partial_i h^{ij} = \delta^{ij} h_{ij} = 0$ .

Let us now discuss the gauge fixing, which is required for the action derivation and the determination of the physical degrees of freedom. The projectability condition of Hořava gravity [3] requires that the perturbation of the lapse-function  $N$  depends only on time, thus  $\phi \equiv \phi(t)$ . This allows us to “gauge away” the  $\phi$ - and  $B$ -perturbations, and also we can eliminate the  $Q_i$  degree of freedom [7]. Therefore, the remaining degrees of freedom are  $\psi$ ,  $E$ ,  $W_i$  and  $h_{ij}$ . In summary, in the aforementioned gauge we obtain

$$\begin{aligned}\delta N &= \delta N_i = 0 \\ \delta_{ij} &= h_{ij} - 2\psi\delta_{ij} + 2\partial_i\partial_j E - (\partial_i W_j + \partial_j W_i).\end{aligned}\quad (24)$$

Note that since only perturbations imposed on the “same-time” spatial hypersurface are allowed, this is equivalent to a *synchronous gauge* choice.

We now perturb the (prior to analytic continuation) Hořava-Lifshitz gravitational action up to second order. After non-trivial but straightforward calculations [7], for the perturbed kinetic part of the action (2) we obtain

$$\delta S_K^{(2)} = \int dt d^3x \frac{2}{\kappa^2} \left[ \frac{1}{4} \dot{h}_{ij} \dot{h}^{ij} + (1-3\lambda) \left( 3\dot{\psi}^2 - 2\dot{\psi} \nabla^2 \dot{E} \right) + (1-\lambda) \dot{E} \nabla^4 \dot{E} \right], \quad (25)$$

while for the perturbed potential part we acquire

$$\begin{aligned}\delta S_V^{(2)} &= \int dt d^3x \left[ \frac{\kappa^2}{8w^4} h_{ij} \nabla^6 h^{ij} + \frac{\kappa^2 \mu}{8w^2} \epsilon^{ijk} h_{ii} \partial_j \nabla^4 h_k^l \right. \\ &\quad - \frac{\kappa^2 \mu^2}{32} h_{ij} \nabla^4 h^{ij} + \frac{\kappa^2 \mu^2 \Lambda}{32(1-3\lambda)} h_{ij} \nabla^2 h^{ij} \\ &\quad - \frac{\kappa^2 \mu^2 (1-\lambda)}{4(1-3\lambda)} \psi \nabla^4 \psi - \frac{\kappa^2 \mu^2 \Lambda}{4(1-3\lambda)} \psi \nabla^2 \psi \\ &\quad + \frac{27\kappa^2 \mu^2 \Lambda^2}{16(1-3\lambda)} \psi^2 - \frac{9\kappa^2 \mu^2 \Lambda^2}{8(1-3\lambda)} \psi \nabla^2 E \\ &\quad \left. + \frac{3\kappa^2 \mu^2 \Lambda^2}{16(1-3\lambda)} E \nabla^4 E \right]. \quad (26)\end{aligned}$$

### A. Scalar perturbations

As can be observed from (25),(26) the action for scalar perturbations includes the two modes  $E$  and  $\psi$ , and their equations of motion read:

$$\frac{8}{\kappa^2} \ddot{E} + \frac{\kappa^2 \mu^2 (1-\lambda)}{2(1-3\lambda)} \nabla^2 \psi + \frac{\kappa^2 \mu^2 \Lambda}{2(1-3\lambda)} \psi = 0 \quad (27)$$

$$\begin{aligned}\frac{8}{\kappa^2} \frac{1-3\lambda}{1-\lambda} \ddot{\psi} - \frac{9\kappa^2 \mu^2 \Lambda^2}{4(1-\lambda)(1-3\lambda)} \psi + \frac{3\kappa^2 \mu^2 \Lambda^2}{4(1-\lambda)(1-3\lambda)} \nabla^2 E \\ + \frac{\kappa^2 \mu^2 (1-\lambda)}{2(1-3\lambda)} \nabla^4 \psi + \frac{\kappa^2 \mu^2 \Lambda}{2(1-3\lambda)} \nabla^2 \psi = 0.\end{aligned}\quad (28)$$

As can be seen these two equations are coupled, not allowing for a straightforward stability investigation. However, we can still acquire information about the stability of the configuration by studying it at high and low momenta. Taking the IR limit of (28), that is considering the low- $k$  behavior, it reduces to

$$\frac{8}{\kappa^2} \frac{1-3\lambda}{1-\lambda} \ddot{\psi} - \frac{9\kappa^2 \mu^2 \Lambda^2}{4(1-\lambda)(1-3\lambda)} \psi = 0. \quad (29)$$

Thus, this decoupled equation acts as a low-momentum equation of motion for the scalar field  $\psi$ . A straightforward observation from (29) is that it leads to a ghost-like behavior, since it leads to the dispersion relation

$$\omega^2 \equiv m^2 = -\frac{9\kappa^4 \mu^2 \Lambda^2}{32(1-3\lambda)^2} < 0, \quad (30)$$

which induces instabilities, regardless of the  $\lambda$ -value and of the sign of the cosmological constant. Now, for high

$k$ , (28) reduces to

$$\frac{8}{\kappa^2} \frac{1-3\lambda}{1-\lambda} \ddot{\psi} + \frac{\kappa^2 \mu^2 (1-\lambda)}{2(1-3\lambda)} \nabla^4 \psi = 0. \quad (31)$$

Therefore, (31) yields the high- $k$  dispersion relation:

$$\omega^2 \equiv \frac{\kappa^4 \mu^2}{16} \left( \frac{1-\lambda}{1-3\lambda} \right)^2 k^4. \quad (32)$$

### B. Tensor perturbations

Let us now examine the tensor perturbations. Their action can be extracted from (25),(26) and therefore the graviton equation of motion writes as

$$\ddot{h}^{ij} - \frac{\kappa^4 \mu^2 \Lambda}{16(1-3\lambda)} \nabla^2 h^{ij} - \frac{\kappa^4}{4w^4} \nabla^6 h^{ij} - \frac{\kappa^4 \mu}{4w^2} \epsilon^{ilk} \partial_l \nabla^4 h_k^j + \frac{\kappa^4 \mu^2}{16} \nabla^4 h^{ij} = 0. \quad (33)$$

Assuming graviton propagation along the  $x^3$  direction, that is  $k_i = k^i = (0, 0, k)$ , the  $h_{ij}$  can be written as usual in terms of the Left and Right polarization components, and thus we derive the two equations for the different polarizations

$$-\omega^2 \tilde{h}_{L,R} + c^2 k^2 \tilde{h}_{L,R} + \frac{\kappa^4 \mu^2}{16} k^4 \tilde{h}_{L,R} \pm \frac{\kappa^4 \mu}{4w^2} k^5 \tilde{h}_{L,R} + \frac{\kappa^4}{4w^4} k^6 \tilde{h}_{L,R} = 0, \quad (34)$$

where the plus and minus branches correspond to Left-handed and Right-handed modes respectively. In this relation we have identified the light speed from the low  $k$  regime as  $c^2 = \kappa^4 \mu^2 \Lambda / [16(1-3\lambda)]$ . The above equation system accepts a non-trivial solution only if the corresponding determinant is zero, which leads to the dispersion relation

$$\omega^2 = c^2 k^2 + \frac{\kappa^4 \mu^2}{16} k^4 \pm \frac{\kappa^4 \mu}{4w^2} k^5 + \frac{\kappa^4}{4w^4} k^6. \quad (35)$$

### C. Beyond Detailed Balance

In order to avoid possible accidental artifacts of the detailed-balance condition, in this subsection we extend the investigation beyond detailed balance. As a demonstration, and without loss of generality, we consider a detailed-balance-breaking term of the form  $\nabla_i R_{jk} \nabla^i R^{jk}$ . Thus, the corresponding contribution to the action will be [7]

$$\delta S_{bdb}^{(2)} = \eta \int dt d^3x \left( -\frac{1}{4} h_{ij} \nabla^6 h^{ij} - 6\psi \nabla^6 \psi \right), \quad (36)$$

where  $\eta$  is an additional parameter. It is straightforward to calculate the modifications that  $S_{bdb}^{(2)}$  brings to the

dispersion relations for scalar and tensor perturbations obtained above (expressions (32) and (35) respectively). The extended dispersion relations read:

$$\omega^2 \sim \frac{\kappa^2 (1-\lambda)^2}{16(1-3\lambda)^2} k^4 - \frac{3\kappa^2 (1-\lambda)}{2(1-3\lambda)} \eta k^6 \quad (37)$$

for scalar perturbations (UV-behavior), and

$$\omega^2 = c^2 k^2 + \frac{\kappa^4 \mu^2}{16} k^4 \pm \frac{\kappa^4 \mu}{4w^2} k^5 + \left( \frac{\kappa^4}{4w^4} - \frac{\kappa^2 \eta}{2} \right) k^6 \quad (38)$$

for tensor perturbations. As was expected, the detailed-balance-breaking term modifies mainly the UV regime of the theory.

### D. Instabilities

Concerning the scalar perturbations, as was mentioned above (29),(31) leads to instabilities. This unstable behavior cannot be cured by simple tricks such as analytic continuation of the form  $\mu \rightarrow i\mu$ ,  $w^2 \rightarrow -iw^2$  [13], since in that case we straightforwardly see that the UV behavior is spoiled (see (32)) and thus instabilities re-emerge at high energies. Even in this case though, we cannot evade the instability coming from the negative mass term, and thus IR instabilities persist as long as we have a non-vanishing cosmological constant. Finally, concerning the tensor sector, from (35) we see that if we desire a well-behaved UV regime we cannot impose the analytic continuation.

Proceeding to the relaxation of the detailed-balance condition, a crucial observation is that the ghost instability of the scalar mode arises from the kinetic term of the action and thus the breaking of detailed balance, which affects the potential term, will not alter the aforementioned scalar-instabilities results.

## VIII. HEALTHY EXTENSIONS OF HOŘAVA-LIFSHITZ GRAVITY

In the previous section we saw that Hořava-Lifshitz gravity in its simple version, with or without the detailed-balance version, suffers from instabilities and pathologies that cannot be cured. It is thus necessary to try to construct suitable extensions that are free of such problems.

A quite general power-counting renormalizable action is [10]:

$$S = S_{kin} + S_1 + S_2 + S_{new}, \quad (39)$$

with

$$S_{kin} = \alpha \int dt d^3x \sqrt{g} N [(K_{ij} K^{ij} - l K^2)]$$

$$S_1 = \int dt d^3x \sqrt{g} N \left[ \gamma_0 \frac{\epsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l_k + \zeta R_{ij} R^{ij} + \eta R^2 + \xi R + \sigma \right]$$



$$S_2 = \int dt d^3x \sqrt{g} N [\beta_0 C_{ij} C^{ij} + \beta_1 R \square R + \beta_2 R^3 \\ + \beta_3 R R_{ij} R^{ij} + \beta_4 R_{ij} R^{ik} R^j{}_k]$$

$$S_{new} = \int dt d^3x \sqrt{g} N [a_1 (a_i a^i) + a_2 (a_i a^i)^2 + a_3 R^{ij} a_i a_j \\ + a_4 R \nabla_i a^i + a_5 \nabla_i a_j \nabla^i a^j + a_6 \nabla^i a_i (a_j a^j) + \dots] . (40)$$

Thus, apart from the known kinetic, detailed-balance and beyond-detailed-balance combinations that constitute the Hořava-Lifshitz gravitational action, in (40) we have added a new combination, based on the term [9]:

$$a_i \equiv \frac{\partial_i N}{N}, \quad (41)$$

which breaks the projectability condition, and the ellipsis in (40) refers to dimension six terms involving  $a_i$  as well as curvatures.

Such a new combination of terms seems to alleviate the problems of Hořava-Lifshitz gravity, although there could still be some ambiguities [41]. Therefore, one should repeat all the investigations of the present work, for this extended version of the theory.

## IX. CONCLUSIONS

In this work we reviewed some general aspects of Hořava-Lifshitz cosmology. Formulating the basic ver-

sion of Hořava-Lifshitz gravity, with or without the detailed-balance condition, we extracted the cosmological equations. We used observational data in order to constrain the parameters of the theory, and amongst others we saw that the running parameter  $\lambda$ , that determines the flow between the IR and the UV, is indeed restricted in a very narrow window around its IR value 1. Through a phase-space analysis we extracted the late-time stable solutions, which are independent of the initial conditions, and we saw that eternal expansion, or bouncing and cyclic behavior, can arise naturally. Formulating the effective dark energy sector we showed that Hořava-Lifshitz cosmology can describe the phantom phase, without the use of a phantom field. However, performing a detailed perturbation analysis, we showed that Hořava-Lifshitz gravity in its basic version, suffers from instabilities. Thus, one should try to construct suitable generalizations, that are free from pathologies, and then repeat all the above steps of cosmological analysis. Such a task proves to be hard, but it is necessary if we desire Hořava-Lifshitz gravity to be a candidate for the description of nature.

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